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Lepton FCNC in Type III seesaw model

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Abstract: In Type III seesaw model, there are tree level flavor changing neutral currents (FCNC) in the lepton sector, due to mixing of charged particles in the leptonic triplet introduced to realize seesaw mechanism, with the usual charged leptons. In this work we study these FCNC effects in a systematic way using available experimental data. Several FCNC processes have been studied before. The new processes considered in this work include: lepton flavor violating processes $\tau \to Pl$, $\tau \to Vl$, $V \to l\bar{l}'$, $P \to l\bar{l}'$, $M \to M'l\bar{l}'$ and muonium-antimuonium oscillation. Results obtained are compared with previous results from $l_i \to l_j l_k \bar{l}_l$, $l_i \to l_j \gamma$, $Z \to l\bar{l}'$ and $\mu - e$ conversion. Our results show that the most stringent constraint on the e-to-τ FCNC effect comes from $\tau \to \pi^0 e$ decay. $\tau \to \rho^0 \mu$ and $\tau \to \pi^0 \mu$ give very stringent constraints on the μ-to-τ FCNC effect, comparable with that obtained from $\tau \to \mu\bar{\mu}\mu$ studied previously. The constraint on the e-to-μ FCNC effect from processes considered in this work is much weaker than that obtained from processes studies previously, in particular that from $\mu - e$ conversion in atomic nuclei. We find that in the canonical seesaw models the FCNC parameters, due to tiny neutrino masses, are all predicted to be much smaller than the constraints obtained here, making such models irrelevant. However, we also find that in certain special circumstances the tiny neutrino masses do not directly constrain the FCNC parameters. In these situations, the constraints from the FCNC studies can still play important roles.

Keywords: Rare Decays, Beyond Standard Model, Neutrino Physics

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JHEP09(2009)027

Contents

1 Introduction

Neutrino oscillation experiments involving neutrinos and antineutrinos coming from astrophysical and terrestrial sources have found compelling evidence that neutrinos have finite but small masses. To accommodate this observation, the minimal standard model (SM) must be extended. Some sensible ways to do this include: (a) Type I seesaw with three heavy right-handed (RH) Majorana neutrinos [1], (b) the use of an electroweak Higgs triplet to directly provide the left-handed (LH) neutrinos with small Majorana masses (Type II seesaw [2]), (c) introducing fermion triplets with zero hypercharge (Type III seesaw [3]), (d) the generation of three Dirac neutrinos through an exact parallel of the SM method of giving mass to charged fermions, and (e) the radiative generation of neutrino masses as per the Zee or Babu models [4]. But in the absence of more experimental data, it is impossible to tell which, if any, of these is actually correct. Different models should be studied using available data or future ones. In this work, we carry out a systematic study of constraints on possible new flavor changing neutral currents (FCNC) in Type III seesaw model.

The fermion triplet Σ in Type III seesaw model transforms under the SM gauge group $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ as (1,3,0). We will assume that there are three copies of such fermion triplets. The model has many interesting features [5], including the possibility of having low seesaw scale of order a TeV to realize leptogenesis [6] and detectable effects at LHC [7, 8] due to the fact that the heavy triplet leptons have gauge interactions being non-trivial under the $\mathrm{SU}(2)_L$ gauge group, and the possibility of having new tree level FCNC

JHEP09(2009)027

interactions in the lepton sector [9–11]. Some of the FCNC effects have been studied, such as $l_i \to l_j \bar{l}_k l_l$, $l_i \to l_j \gamma$, $Z \to l_i \bar{l}_j$ and $\mu - e$ conversion processes. Several other FCNC processes studied experimentally have not been studied in the context of Type III seesaw model. We will study constraints on FCNC related to charged leptons in a systematic way using available experimental bounds listed in ref. [12] by the particle data group.

Before studying constraints, let us describe the model in more detail to identify new tree level FCNC in the lepton sector. The component fields of the righthanded triplet Σ are,

$$\Sigma = \begin{pmatrix} N^0/\sqrt{2} & E^+ \\ E^- & -N^0/\sqrt{2} \end{pmatrix}, \quad \Sigma^c = \begin{pmatrix} N^{0c}/\sqrt{2} & E^{-c} \\ E^{+c} & -N^{0c}/\sqrt{2} \end{pmatrix}, \tag{1.1}$$

and the renormalizable Lagrangian involving Σ is given by

$$\mathcal{L} = Tr\left[\overline{\Sigma} i\not{D}\Sigma\right] - \frac{1}{2} Tr\left[\overline{\Sigma} M_\Sigma \Sigma^c + \overline{\Sigma^c} M_\Sigma^* \Sigma\right] - \tilde{H}^\dagger \overline{\Sigma}\sqrt{2} Y_\Sigma L_L - \overline{L_L}\sqrt{2} Y_\Sigma{}^\dagger \Sigma \tilde{H}\,,$$

where $L_L = (\nu_L, l_L^-)^T$ is the lepton doublet. $H \equiv (\phi^+, \phi^0)^T \equiv (\phi^+, (v + h + i\eta)/\sqrt{2})^T$ is the Higgs doublet with v being the vacuum expectation value, and $\tilde{H} = i\tau_2 H^*$.

Defining $E \equiv E_R^{+c} + E_R^-$ and removing the would-be Goldstone bosons η and $\phi^\pm$, one obtains the Lagrangian

$$\begin{aligned} \mathcal{L} = {} & \overline{E} i\not{\partial} E + \overline{N_R^0} i\not{\partial} N_R^0 - \overline{E} M_\Sigma E - \left(\overline{N_R^0}\frac{M_\Sigma}{2} N_R^{0c} + \text{h.c.}\right) \\ & + g\left(W_\mu^+ \overline{N_R^0}\gamma_\mu P_R E + W_\mu^+ \overline{N_R^{0c}}\gamma_\mu P_L E + \text{h.c.}\right) - g\, W_\mu^3 \overline{E}\gamma_\mu E \\ & - \left(\frac{1}{\sqrt{2}}(v+h)\overline{N_R^0} Y_\Sigma \nu_L + (v+h)\overline{E} Y_\Sigma l_L + \text{h.c.}\right). \end{aligned} \tag{1.2}$$

One can easily identify the terms related to neutrino masses from the above. The mass matrix is the seesaw form

$$\mathcal{L} = -(\overline{\nu_L^c}\ \overline{N^0}) \begin{pmatrix} 0 & {Y_\Sigma}^T v/2\sqrt{2} \\ Y_\Sigma v/2\sqrt{2} & M_\Sigma/2 \end{pmatrix} \begin{pmatrix} \nu_L \\ N^{0c} \end{pmatrix} + \text{h.c.}\,. \tag{1.3}$$

The charged partners in the triplets mix with the SM charged leptons resulting in a mass matrix of the following form

$$\mathcal{L} = -(\overline{l_R}\ \overline{E_R}) \begin{pmatrix} m_l & 0 \\ Y_\Sigma v & M_\Sigma \end{pmatrix} \begin{pmatrix} l_L \\ E_L \end{pmatrix} + \text{h.c.}\,. \tag{1.4}$$

One can diagonalize the fermion mass matrices and find the transformation matrices between fields in weak interaction basis and in mass eigenstate basis defined as

$$\begin{pmatrix} l_{L,R} \\ E_{L,R} \end{pmatrix} = U_{L,R} \begin{pmatrix} l'_{L,R} \\ E'_{L,R} \end{pmatrix}, \quad \begin{pmatrix} \nu_L \\ N^{0c} \end{pmatrix} = U \begin{pmatrix} \nu'_L \\ N'^{0c} \end{pmatrix},$$

JHEP09(2009)027

where the primed fields indicate mass eigenstates. $U_{L,R}$ are $(3+3)$-by-$(3+3)$ matrices if 3 triplets are present, and can be written as

$$U_L \equiv \begin{pmatrix} U_{Lll} & U_{LlE} \\ U_{LEl} & U_{LEE} \end{pmatrix}, U_R \equiv \begin{pmatrix} U_{Rll} & U_{RlE} \\ U_{REl} & U_{REE} \end{pmatrix}, U \equiv \begin{pmatrix} U_{\nu\nu} & U_{\nu N} \\ U_{N\nu} & U_{NN} \end{pmatrix}. \tag{1.5}$$

To order $v^2 M_\Sigma^{-2}$, one has [9]

$$\begin{aligned} U_{Lll} &= 1-\epsilon\,, \quad U_{LlE} = Y_\Sigma^\dagger M_\Sigma^{-1} v\,, & U_{LEl} &= -M_\Sigma^{-1} Y_\Sigma v\,, & U_{LEE} &= 1-\epsilon'\,, \\ U_{Rll} &= 1\,, \quad U_{RlE} = m_l Y_\Sigma^\dagger M_\Sigma^{-2} v\,, & U_{REl} &= -M_\Sigma^{-2} Y_\Sigma m_l v\,, & U_{LEE} &= 1\,, \\ U_{\nu\nu} &= (1-\epsilon/2) U_{PMNS}\,, & U_{\nu N} &= Y_\Sigma^\dagger M_\Sigma^{-1} v/\sqrt{2}\,, & & \\ U_{N\nu} &= -M_\Sigma^{-1} Y_\Sigma U_{\nu\nu} v/\sqrt{2}\,, & U_{NN} &= 1-\epsilon'/2\,, & & \end{aligned} \tag{1.6}$$

where

$$\epsilon = Y_\Sigma^\dagger M_\Sigma^{-2} Y_\Sigma v^2/2 = U_{\nu N} U_{\nu N}^\dagger \qquad \epsilon' = M_\Sigma^{-1} Y_\Sigma Y_\Sigma^\dagger M_\Sigma^{-1} v^2/2 = U_{\nu N}^\dagger U_{\nu N}\,. \tag{1.7}$$

Here U_{PMNS} denotes the lowest order Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix which is unitary. We have kept higher order corrections to the $U_{\nu\nu}$ matrix.

Using the above, one obtains the couplings of Z and physical Higgs h to the usual charged leptons

$$\begin{aligned} \mathcal{L}_{NC} &= \frac{g}{cos\theta_W} \bar{l} \gamma^\mu \left(P_L \left(-\frac{1}{2} + \sin^2\theta_W - \epsilon \right) + P_R \sin^2\theta_W \right) l Z_\mu\,, \\ \mathcal{L}_H &= \frac{g}{2M_W} \bar{l} \left(P_L m_l (3\epsilon - 1) + P_R (3\epsilon - 1) m_l \right) l h\,. \end{aligned} \tag{1.8}$$

Here we have dropped the "prime" on the fermion mass eigenstates. ϵ is a 3-by-3 matrix. Non-zero off diagonal elements in ϵ are the new sources of tree level FCNC in charged lepton sector. The Z and Higgs coupling to quarks are the same as in the SM. We will use available FCNC data in a systematic way to constrain the parameter $\epsilon_{ll'}$.

Several processes, such as $l_i \to l_j l_k \bar{l}_l$, $l_i \to l_j \gamma$, $Z \to l\bar{l}'$ and $\mu - e$ conversion in atomic nuclei, have been studied and stringent constraints have been obtained for $\epsilon_{ll'}$ which will be used as standards for constraints obtained from new lepton flavor violating (LFV) processes considered here, $\tau \to Pl$, $\tau \to Vl$, $V \to l\bar{l}'$, $P \to l\bar{l}$, $M \to M' l\bar{l}'$ and muonium-antimuonium oscillation. It turns out that with currently available experimental data, the LFV processes considered in this work involving τ leptons provide very stringent constraints on the FCNC parameter $\epsilon_{i\tau}$. Our results show that the most stringent constraint on $\epsilon_{e\tau}$ comes from $\tau \to \pi^0 e$ decay. $\tau \to \rho^0 \mu$ and $\tau \to \pi^0 \mu$ give very stringent constraints on $\epsilon_{\mu\tau}$, comparable with that obtained from $\tau \to \mu\bar{\mu}\mu$ in previous studies. The strongest constraint on $\epsilon_{e\mu}$ comes from $\mu - e$ conversion in atomic nuclei studied previously. We now present some details for the new processes mentioned above.

2 Constraints from $\tau \to Pl$ and $\tau \to Vl$

Exchange of Z boson between quarks and leptons can induce $\tau \to Pl$ and $\tau \to Vl$ at tree level, where a pseudoscalar meson $P = \pi^0, \eta, \eta'$ and a vector meson $V = \rho^0, \omega, \phi$ and a

charged lepton $l = e, \mu$. The decay amplitudes for $\tau \to Ml$ (where M denotes either V or P) can be written in the following form

$$\begin{aligned}\mathcal{M} &= 2\sqrt{2}G_F\ \epsilon_{l\tau} \sum_{q=u,d,s} \langle M(p_{_M})|\bar{q}\gamma_\alpha(I_3 P_L - Q_q \sin^2\theta_W)q|0\rangle \cdot \left[\bar{l}(p_{_l})\gamma^\alpha(1-\gamma_5)\tau(p_\tau)\right] \\ &= 2\sqrt{2}G_F\ \epsilon_{l\tau} \sum_{q=u,d,s} \langle M(p_{_M})|\bar{q}\gamma_\alpha(g_{_V}^q + g_{_A}^q\gamma_5)q|0\rangle \cdot \left[\bar{l}(p_{_l})\gamma^\alpha(1-\gamma_5)\tau(p_\tau)\right], \end{aligned} \tag{2.1}$$

where G_F is the Fermi constant, Q_q is the electric charge of q-quark in unit of proton charge. $I_3 = 1/2$ and $-1/2$ for up and down type of quarks, respectively. The factor $g_{_V}^q = \frac{1}{4} - \frac{2}{3}\sin^2\theta_W$ and $g_{_A}^q = -\frac{1}{4}$ for up type of quarks, and $g_{_V}^q = -\frac{1}{4} + \frac{1}{3}\sin^2\theta_W$ and $g_{_A}^q = \frac{1}{4}$ for down type of quarks. The p_τ, $p_{_l}$ and $p_{_M}$ are the momenta of τ, l and M, respectively.

For $\tau^- \to \pi^0 l$, the decay constant f_π is defined as

$$\langle \pi^0(p_\pi)|\bar{u}\gamma_\alpha\gamma_5 u|0\rangle = -\langle \pi^0(p_\pi)|\bar{d}\gamma_\alpha\gamma_5 d|0\rangle = -i\frac{f_\pi}{\sqrt{2}}(p_\pi)_\alpha \tag{2.2}$$

and its value is $f_\pi = 130.4$ MeV. For $\tau^- \to \eta l$ and $\tau^- \to \eta' l$, due to the $\eta - \eta'$ mixing, the decay constants $f_{\eta^{(\prime)}}^u$ and $f_{\eta^{(\prime)}}^s$ are defined as

$$\begin{aligned}\langle \eta^{(\prime)}(p_{\eta^{(\prime)}})|\bar{u}\gamma_\alpha\gamma_5 u|0\rangle &= \langle \eta^{(\prime)}(p_{\eta^{(\prime)}})|\bar{d}\gamma_\alpha\gamma_5 d|0\rangle = -if_{\eta^{(\prime)}}^u(p_{\eta^{(\prime)}})_\alpha, \\ \langle \eta^{(\prime)}(p_{\eta^{(\prime)}})|\bar{s}\gamma_\alpha\gamma_5 s|0\rangle &= -if_{\eta^{(\prime)}}^s(p_{\eta^{(\prime)}})_\alpha, \end{aligned} \tag{2.3}$$

where

$$\begin{aligned} f_\eta^u &= \frac{f_8}{\sqrt{6}}\cos\theta_8 - \frac{f_0}{\sqrt{3}}\sin\theta_0, \qquad f_\eta^s = -2\frac{f_8}{\sqrt{6}}\cos\theta_8 - \frac{f_0}{\sqrt{3}}\sin\theta_0, \\ f_{\eta'}^u &= \frac{f_8}{\sqrt{6}}\sin\theta_8 + \frac{f_0}{\sqrt{3}}\cos\theta_0, \qquad f_{\eta'}^s = -2\frac{f_8}{\sqrt{6}}\sin\theta_8 + \frac{f_0}{\sqrt{3}}\cos\theta_0, \end{aligned} \tag{2.4}$$

with $f_8 = 168$ MeV, $f_0 = 157$ MeV, $\theta_8 = -22.2^\circ$, and $\theta_0 = -9.1^\circ$ [13].

For $\tau^- \to Vl$ decays, the decay constants f_ρ, f_ω and f_ϕ are defined by

$$\begin{aligned}\langle \rho^0(p_\rho)|\bar{u}\gamma_\alpha u|0\rangle &= -\langle \rho^0(p_\rho)|\bar{d}\gamma_\alpha d|0\rangle = \frac{f_\rho}{\sqrt{2}}m_\rho(\epsilon_\rho)_\alpha, \\ \langle \omega(p_\omega)|\bar{u}\gamma_\alpha u|0\rangle &= \langle \omega(p_\omega)|\bar{d}\gamma_\alpha d|0\rangle = \frac{f_\omega}{\sqrt{2}}m_\omega(\epsilon_\omega)_\alpha, \\ \langle \phi(p_\phi)|\bar{s}\gamma_\alpha s|0\rangle &= f_\phi m_\phi(\epsilon_\phi)_\alpha, \end{aligned} \tag{2.5}$$

where $(\epsilon_{_V})_\alpha$ is the polarization vector of V. We use $f_\rho = 205$ MeV, $f_\omega = 195$ MeV and $f_\phi = 231$ MeV [14].

Exchange of Higgs boson can also induce $q\bar{q}$ coupling to $l\bar{l}'$. However, Higgs-mediated diagrams do not contribute to $\tau \to Pl$ and $\tau \to Vl$ because the bi-quark operator in this case is of the form $\bar{q}q$ which induces a vanishing matrix element for $< P$ (or $V)|\bar{q}q|0 >$.

The decay rate for $\tau^- \to Pl$ ($P = \pi^0, \eta, \eta'$, and $l = e,\ \mu$), averaged over the spin of τ and summed over the spin of l, is given by

$$\Gamma = a_P\frac{G_F^2 f_P^2}{2\pi m_\tau^2}\ |\epsilon_{l\tau}|^2\ |\vec{p}_{_l}|\left[m_\tau^4 + m_{_l}^4 - 2m_{_l}^2 m_\tau^2 - (m_{_l}^2 + m_\tau^2)m_{_P}^2)\right], \tag{2.6}$$

Process	Branching Ratio	Constraint on $\lvert\epsilon_{ll'}\rvert$
$\tau^- \to \pi^0 e^-$	$< 8.0 \times 10^{-8}$	$\lvert\epsilon_{e\tau}\rvert < 4.2 \times 10^{-4}$
$\tau^- \to \pi^0 \mu^-$	$< 1.1 \times 10^{-7}$	$\lvert\epsilon_{\mu\tau}\rvert < 7.0 \times 10^{-4}$
$\tau^- \to \eta e^-$	$< 9.2 \times 10^{-8}$	$\lvert\epsilon_{e\tau}\rvert < 1.2 \times 10^{-3}$
$\tau^- \to \eta \mu^-$	$< 6.5 \times 10^{-8}$	$\lvert\epsilon_{\mu\tau}\rvert < 9.7 \times 10^{-4}$
$\tau^- \to \eta' e^-$	$< 1.6 \times 10^{-7}$	$\lvert\epsilon_{e\tau}\rvert < 1.0 \times 10^{-3}$
$\tau^- \to \eta' \mu^-$	$< 1.3 \times 10^{-7}$	$\lvert\epsilon_{\mu\tau}\rvert < 1.0 \times 10^{-3}$
$\tau^- \to \rho^0 e^-$	$< 6.3 \times 10^{-8}$	$\lvert\epsilon_{e\tau}\rvert < 6.5 \times 10^{-4}$
$\tau^- \to \rho^0 \mu^-$	$< 6.8 \times 10^{-8}$	$\lvert\epsilon_{\mu\tau}\rvert < 6.8 \times 10^{-4}$
$\tau^- \to \omega e^-$	$< 1.1 \times 10^{-7}$	$\lvert\epsilon_{e\tau}\rvert < 3.2 \times 10^{-3}$
$\tau^- \to \omega \mu^-$	$< 8.9 \times 10^{-8}$	$\lvert\epsilon_{\mu\tau}\rvert < 2.5 \times 10^{-3}$
$\tau^- \to \phi e^-$	$< 7.3 \times 10^{-8}$	$\lvert\epsilon_{e\tau}\rvert < 7.5 \times 10^{-4}$
$\tau^- \to \phi \mu^-$	$< 1.3 \times 10^{-7}$	$\lvert\epsilon_{\mu\tau}\rvert < 1.0 \times 10^{-3}$

Table 1. Constraints from $\tau \to Pl$ and $\tau \to Vl$.

where $|\vec{p}_l| = \sqrt{(m_\tau^2 + m_P^2 - m_l^2)^2 - 4m_\tau^2 m_P^2}/(2m_\tau)$. In the above expression, the decay constant f_P is given by $f_P = f_\pi$ with $a_P = 1$ for $\tau^- \to \pi^0 l$, and $f_P = f^s_{\eta^{(\prime)}}$ with $a_P = 1/2$ for $\tau^- \to \eta^{(\prime)} l$. In the case of $\tau^- \to \eta^{(\prime)} l$, the u and d quark contributions to the matrix element $\langle \eta^{(\prime)} | \bar{q}\gamma_\alpha\gamma_5 q|0\rangle$ cancel each other in eq. (2.1) so that only the s quark contribution to the decay constant, $f^s_{\eta^{(\prime)}}$, remains.

Similarly, the decay rate for $\tau^- \to Vl$ ($P = \rho^0, \omega, \phi$, and $l = e,\ \mu$) is given by

$$\Gamma = a_V \frac{G_F^2 f_V^2 m_V^2}{\pi m_\tau^2}\ |\epsilon_{l\tau}|^2\, |\vec{p}_l| \left[m_\tau^2 + m_l^2 - m_V^2 + \frac{1}{m_V^2}(m_\tau^2 + m_V^2 - m_l^2)(m_\tau^2 - m_V^2 - m_l^2) \right], \quad (2.7)$$

where $|\vec{p}_l| = \sqrt{(m_\tau^2 + m_V^2 - m_l^2)^2 - 4m_\tau^2 m_V^2}/(2m_\tau)$. The decay constant f_V is given by $f_V = f_\rho$ with $a_V = (1/2 - \sin^2\theta_W)^2$ for $\tau^- \to \rho^0 l$, and $f_V = f_\omega$ with $a_V = (\sin^2\theta_W/3)^2$ for $\tau^- \to \omega l$, and $f_V = f_\phi$ with $a_V = 2(1/4 - \sin^2\theta_W/3)^2$ for $\tau^- \to \phi l$.

Using the current experimental bounds on the branching ratios, we find the constraints on the parameters $|\epsilon_{e\tau}|$ and $|\epsilon_{\mu\tau}|$ which are shown in table 1. Notice that the constraint on $|\epsilon_{e\tau}|$ obtained from $\tau^- \to \pi^0 e^-$ is $|\epsilon_{e\tau}| < 4.2 \times 10^{-4}$, which is more stringent than the so far most stringent bound obtained from $\tau \to e\bar{e}e$ as shown in table 4. The constraints on $|\epsilon_{\mu\tau}|$ obtained from $\tau^- \to \pi^0\mu^-$ and $\tau^- \to \rho^0\mu^-$ are comparable to the so far most stringent bound shown in table 4. The upper bounds on $|\epsilon_{e(\mu)\tau}|$ from $\tau^- \to \eta^{(\prime)} l$ and $\tau^- \to \omega l$ are weaker.

3 Constraints from $V \to l\bar{l}'$ and $P \to l\bar{l}'$

Here V can be a vector meson J/ψ or Υ, and P can be a pseudoscalar meson π^0, η or η'. The l and l' stand for charged leptons with different flavors $l \neq l'$. These processes can be induced by exchange Z boson between quarks and leptons. The general decay amplitude

JHEP09(2009)027

Process	Branching Ratio	Constraint on $\lvert\epsilon_{ll'}\rvert$
$\Upsilon(3S) \to e^{\pm}\tau^{\mp}$	$< 5 \times 10^{-6}$	$\lvert\epsilon_{e\tau}\rvert < 0.39$
$\Upsilon(3S) \to \mu^{\pm}\tau^{\mp}$	$< 4.1 \times 10^{-6}$	$\lvert\epsilon_{\mu\tau}\rvert < 0.35$
$J/\Psi(1S) \to e^{\pm}\mu^{\mp}$	$< 1.1 \times 10^{-6}$	$\lvert\epsilon_{e\mu}\rvert \sim O(1)$
$J/\Psi(1S) \to e^{\pm}\tau^{\mp}$	$< 8.3 \times 10^{-6}$	$\lvert\epsilon_{e\tau}\rvert \sim O(1)$
$J/\Psi(1S) \to \mu^{\pm}\tau^{\mp}$	$< 2.0 \times 10^{-6}$	$\lvert\epsilon_{\mu\tau}\rvert \sim O(1)$
$\pi^0 \to e^{+}\mu^{-}$	$< 3.4 \times 10^{-9}$	$\lvert\epsilon_{e\mu}\rvert < 0.80$
$\pi^0 \to e^{-}\mu^{+}$	$< 3.8 \times 10^{-10}$	$\lvert\epsilon_{e\mu}\rvert < 0.27$
$\eta \to e^{\pm}\mu^{\mp}$	$< 6 \times 10^{-6}$	$\lvert\epsilon_{e\mu}\rvert \sim O(1)$
$\eta' \to e^{\pm}\mu^{\mp}$	$< 4.7 \times 10^{-4}$	$\lvert\epsilon_{e\mu}\rvert \sim O(1)$

Table 2. Constraints from $V \to l\bar{l}'$ and $P \to l\bar{l}'$.

for $M \to l\bar{l}'$ (where M denotes either V or P) is given by

$$\begin{aligned}\mathcal{M} &= 2\sqrt{2}G_F\ \epsilon_{ll'} \sum_{q=u,d,s,c,b} \langle 0|\bar{q}\gamma_\alpha(I_3 P_L - Q_q \sin^2\theta_W)q|M(p_M)\rangle \cdot [\bar{u}_l(p_1)\gamma^\alpha(1-\gamma_5)v_{l'}(p_2)] \\ &= 2\sqrt{2}G_F\ \epsilon_{ll'} \sum_{q=u,d,s,c,b} \langle 0|\bar{q}\gamma_\alpha(g_V^q + g_A^q\gamma_5)q|M(p_M)\rangle \cdot [\bar{u}_l(p_1)\gamma^\alpha(1-\gamma_5)v_{l'}(p_2)]\,, \end{aligned} \tag{3.1}$$

where we use the decay constants $f_{J/\Psi} = 416$ MeV and $f_{\Upsilon(3S)} = 430$ MeV [12, 15]. Again, exchange of Higgs boson does not contribute to these two classes of processes since $\langle 0|\bar{q}q|M\rangle = 0$.

The decay rate for $V \to l\bar{l}'$ ($V = J/\Psi, \Upsilon$) is found to be

$$\Gamma = \frac{8G_F^2 f_V^2}{3\pi}(g_V^q)^2|\epsilon_{ll'}|^2\ |\vec{p}_l|\left[m_V^2 - \frac{1}{2}m_l - \frac{1}{2}m_{l'} - \frac{1}{2m_V^2}(m_l^2 - m_{l'}^2)^2\right], \tag{3.2}$$

where $|\vec{p}_l| = \sqrt{(m_V^2 + m_l^2 - m_{l'}^2)^2 - 4m_V^2 m_l^2}/(2m_V)$, and $g_V^q = g_V^c$ for $V = J/\Psi$ and $g_V^q = g_V^b$ for $V = \Upsilon$.

Similarly the rate of a pseudoscalar meson decay $P \to l\bar{l}'$ ($P = \pi^0, \eta, \eta'$) is given by

$$\Gamma = a_P \frac{G_F^2 f_P^2}{2\pi m_P}|\epsilon_{ll'}|^2\ |\vec{p}_l|\left[(m_l^2 + m_{l'}^2)m_P^2 - (m_l^2 - m_{l'}^2)^2\right], \tag{3.3}$$

where $|\vec{p}_l| = \sqrt{(m_P^2 + m_l^2 - m_{l'}^2)^2 - 4m_P^2 m_l^2}/(2m_P)$, and $a_P = 1$, $f_P = f_\pi$ for $P = \pi^0$, and $a_P = 1/2$, $f_P = f_{\eta^{(\prime)}}^s$ for $P = \eta^{(\prime)}$. Note that as in the case of $\tau^- \to \eta^{(\prime)}l$, only the s quark contribution to the decay constant, $f_{\eta^{(\prime)}}^s$, appears in $\eta^{(\prime)} \to l\ \bar{l}'$. We find that the constraints on $|\epsilon_{ll'}|$ from these two body meson decays are rather weak as summarized in table 2. The constraints obtained are much weaker than those obtained in the previous section.

4 Constraints from $M \to M'l\bar{l}'$

We now consider semileptonic three body decays of the type $M \to M'l\bar{l}'$ with $M = B, K$ and $M' = K, K^*, \pi$, such as $B \to Kl\bar{l}'$, $B \to K^*l\bar{l}'$, $B \to \pi l\bar{l}'$, and $K \to \pi l\bar{l}'$. These decays

JHEP09(2009)027

can occur through quark level subprocesses $b \to s l\bar{l}'$ or $s \to d l\bar{l}'$. The FCNC $b \to s$ or $s \to d$ transition can arise via Z-penguin and Higgs-penguin diagrams at one loop level the same way as in the SM. After taking into account the SM effective b-s-Z and b-s-Higgs couplings (or s-d-Z and s-d-Higgs couplings) [16, 17], the lepton flavor violating FCNC processes $b \to s l\bar{l}'$ (or $s \to d l\bar{l}'$) can occur at tree level via the couplings given in eq. (1.8).

The decay amplitude for $M \to M' l\bar{l}'$ [where $M = B$ (or K); $M' = K, K^*, \pi$ (or π); $l,\ l' = e,\ \mu,\ \tau\ (l \neq l')$)] is given by

$$\mathcal{M} = \mathcal{M}^Z + \mathcal{M}^h, \tag{4.1}$$

where $\mathcal{M}^Z$ and $\mathcal{M}^h$ denote the Z-mediated and Higgs-mediated decay amplitude, respectively, in the following form

$$\begin{aligned}
\mathcal{M}^Z &= -\frac{1}{32\pi^2} V^*_{iq''} V_{iq'} \frac{g^4}{\cos^2\theta_W M_W^2}\, C_0(x_i)\, \epsilon_{ll'}\, \langle M'(p')|\bar{q}''\gamma_\alpha(1-\gamma_5)q'|M(p)\rangle \\
&\quad \times [\bar{u}_l(k_1)\gamma^\alpha(1-\gamma_5)v_{l'}(k_2)]\,, \qquad (4.2)\\
\mathcal{M}^h &= i\frac{9}{1024\pi^2} V^*_{tq''} V_{tq'} g^4 \frac{m_t^2 m_{q'}}{m_W^4 m_h^2}\, \epsilon_{ll'}\, \langle M'(p')|\bar{q}''(1+\gamma_5)q'|M(p)\rangle \\
&\quad \times \{\bar{u}_l(k_1)[(m_l + m_{l'}) + (m_{l'} - m_l)\gamma_5]v_{l'}(k_2)\}\,, \qquad (4.3)
\end{aligned}$$

where (i) for $B \to K^{(*)} l\bar{l}'$, $q' = b$ and $q'' = s$, (ii) for $B \to \pi l\bar{l}'$, $q' = b$ and $q'' = d$, (iii) for $K \to \pi l\bar{l}'$, $q' = s$ and $q'' = d$. The $V_{iq'}$ denotes the CKM matrix element with $i = t, c, u$ and $C_0(x_i) = (x_i/8)\left[(x_i-6)/(x_i-1) + (3x_i+2)\ln x_i/(x_i-1)^2\right]$ with $x_i = m_i^2/m_W^2$ [16].

Compared with the Z-mediated amplitude, the Higgs-mediated amplitude is negligibly small, since $m_h \gg m_b, m_l$, so that the Higgs contribution can be safely neglected. For example, in the cases of $B \to K^{(*)} l\bar{l}'$ and $K \to \pi l\bar{l}'$ decays, $\left|\mathcal{M}^h/\mathcal{M}^Z\right|$ is suppressed roughly by $O(x_t(m_b m_l/m_h^2))$ and $O(x_t(m_s m_l/m_h^2))$, respectively.

For $B \to P l\bar{l}'$ ($P = \pi, K$), the form factors F_1 and F_0 (or f_+ and f_-) are defined by

$$\begin{aligned}
\langle P(p')|\bar{s}\gamma_\alpha(1-\gamma_5)b|B(p)\rangle &= F_1(q^2)\left[(p+p')_\alpha - \frac{m_B^2 - m_K^2}{q^2}q_\alpha\right] + F_0(q^2)\frac{m_B^2 - m_K^2}{q^2}q_\alpha \\
&= f_+(q^2)(p+p')_\alpha + f_-(q^2)q_\alpha, \qquad (4.4)
\end{aligned}$$

where $q \equiv p - p'$. For $B \to K^* l\bar{l}'$, the form factors V, A_0, A_1, and A_2 are defined by

$$\begin{aligned}
\langle K^*(p',\epsilon)|\bar{s}\gamma_\alpha(1-\gamma_5)b|B(p)\rangle &= -\epsilon_{\alpha\beta\rho\sigma}\epsilon^{\beta *} p^\rho p'^\sigma \frac{2V(q^2)}{m_B + m_{K^*}} \\
&\quad -i\left(\epsilon^*_\alpha - \frac{\epsilon^*\cdot q}{q^2}q_\alpha\right)(m_B + m_{K^*})A_1(q^2) \\
&\quad +i\left((p+p')_\alpha - \frac{m_B^2 - m_{K^*}^2}{q^2}q_\alpha\right)(\epsilon^*\cdot q)\frac{A_2(q^2)}{m_B + m_{K^*}} \\
&\quad -i\frac{2m_{K^*}(\epsilon^*\cdot q)}{q^2}q_\alpha A_0(q^2), \qquad (4.5)
\end{aligned}$$

where ϵ is the polarization vector of the K^* meson. For numerical analysis, we use the form factors calculated in the framework of light-cone QCD sum rules [18]. The q^2 dependence

JHEP09(2009)027

of the form factors can be expressed as

$$F(q^2) = \frac{F(0)}{1 - a_F \frac{q^2}{m_B^2} + b_F \left(\frac{q^2}{m_B^2}\right)^2}, \tag{4.6}$$

where the values of the parameters $F(0)$, a_F and b_F for $B \to \pi$, $B \to K$ and $B \to K^*$ are given in [18].

Summing over the spins of the final leptons, we obtain

$$\begin{aligned}\frac{d\Gamma(B \to P l \bar{l}')}{dq^2} = \frac{1}{192\pi^5} \frac{G_F^2 \alpha^2}{\sin^4\theta_W \cos^4\theta_W} |V_{ts}^* V_{tb}|^2 C_0^2(x_t) |\epsilon_{ll'}|^2 \frac{\lambda^{3/2}(m_B^2, m_P^2, q^2)}{m_B^3} \\ \times (1-2\rho)^2 \left[(1+\rho)\left|f_+(q^2)\right|^2 + 3\rho \left|f_0(q^2)\right|^2\right],\end{aligned} \tag{4.7}$$

where $\lambda(a,b,c) = (a-b-c)^2 - 4bc$, $\rho = m_l/(2q^2)$ and

$$f_0(q^2) \equiv \frac{(m_B^2 - m_P^2) f_+(q^2) + q^2 f_-(q^2)}{\lambda^{1/2}(m_B^2, m_P^2, q^2)}. \tag{4.8}$$

Here the mass of only one light lepton in the final state has been neglected so that the parameter ρ represents the effect of the remaining lepton mass, e.g. m_τ. Thus, for $B \to K e \mu$ decays, ρ can be neglected. The decay rate for $B \to K^* l \bar{l}'$, summed over the spins of the final leptons and K^*, is given by

$$\begin{aligned}\frac{d\Gamma(B \to K^* l \bar{l}')}{ds} = \frac{1}{768\pi^5} \frac{G_F^2 \alpha^2}{\sin^4\theta_W \cos^4\theta_W} |V_{ts}^* V_{tb}|^2 C_0^2(x_t) |\epsilon_{ll'}|^2 \, m_B^3 \tilde{\lambda}^{1/2} \\ \times \Bigg\{ \left|V(q^2)\right|^2 \frac{8 m_B^4 s \tilde{\lambda}}{(m_B + m_{K^*})^2} \\ + \left|A_1(q^2)\right|^2 (m_B + m_{K^*})^2 \left(\frac{\tilde{\lambda}}{r} + 12 s\right) \\ + \left|A_2(q^2)\right|^2 \frac{m_B^4}{(m_B + m_{K^*})^2} \frac{\tilde{\lambda}^2}{r} \\ - 2 m_B^2 \, \mathrm{Re}\left[A_1(q^2) A_2^*(q^2)\right] \frac{\tilde{\lambda}(1 - r - s)}{r} \Bigg\},\end{aligned} \tag{4.9}$$

where $r = m_{K^*}^2/m_B^2$, $s = q^2/m_B^2$, and $\tilde{\lambda} = 1 + r^2 + s^2 - 2r - 2s - 2rs$. The branching ratios for $B \to \pi l \bar{l}'$, $B \to K l \bar{l}'$ and $B \to K^* l \bar{l}'$ can be calculated after the decay rates given in eqs. (4.7) and (4.9) are integrated in the range $(m_l + m_{l'})^2 \le q^2 \le (m_B - m_{M'})^2$. From the current experimental bounds on those branching ratios, we obtain the constraints on $\epsilon_{ll'}$ shown in table 3.

For $K \to \pi l \bar{l}'$, we normalize the branching ratio to $K^+ \to \pi^0 e^+ \nu_e$ and neglect the phase factor difference [19]. We have

$$\begin{aligned}\frac{B(K^+ \to \pi^+ l \bar{l}')}{B(K^+ \to \pi^0 e^+ \nu_e)} &= \frac{2\alpha^2}{\pi^2 \sin^4\theta_W \cos^4\theta_W} \left|\frac{V_{ts}^* V_{td}}{V_{us}}\right|^2 C_0^2(x_t) \, |\epsilon_{ll'}|^2 , \\ \frac{B(K_L \to \pi^0 l \bar{l}')}{B(K^+ \to \pi^0 e^+ \nu_e)} &= \frac{\tau_{K_L}}{\tau_{K^+}} \frac{2\alpha^2}{\pi^2 \sin^4\theta_W \cos^4\theta_W} \left|\mathrm{Im}\left(\frac{V_{ts}^* V_{td}}{V_{us}}\right)\right|^2 C_0^2(x_t) \, |\epsilon_{ll'}|^2 ,\end{aligned} \tag{4.10}$$

JHEP09(2009)027

Process	Branching Ratio	Constraint on $\lvert\epsilon_{ll'}\rvert$
$B^+ \to \pi^+ e^+ \mu^-$	$< 6.4 \times 10^{-3}$	$\lvert\epsilon_{e\mu}\rvert \sim O(1)$
$B^+ \to \pi^+ e^- \mu^+$	$< 6.4 \times 10^{-3}$	$\lvert\epsilon_{e\mu}\rvert \sim O(1)$
$B^+ \to \pi^+ e^\pm \mu^\mp$	$< 1.7 \times 10^{-7}$	$\lvert\epsilon_{e\mu}\rvert < 0.56$
$B^+ \to K^+ e^+ \mu^-$	$< 9.1 \times 10^{-8}$	$\lvert\epsilon_{e\mu}\rvert < 0.18$
$B^+ \to K^+ e^- \mu^+$	$< 1.3 \times 10^{-7}$	$\lvert\epsilon_{e\mu}\rvert < 0.21$
$B^+ \to K^+ e^\pm \mu^\mp$	$< 9.1 \times 10^{-8}$	$\lvert\epsilon_{e\mu}\rvert < 0.12$
$B^+ \to K^+ \mu^\pm \tau^\mp$	$< 7.7 \times 10^{-5}$	$\lvert\epsilon_{\mu\tau}\rvert \sim O(1)$
$B^0 \to \pi^0 e^\pm \mu^\mp$	$< 1.4 \times 10^{-7}$	$\lvert\epsilon_{e\mu}\rvert < 0.73$
$B^0 \to K^0 e^\pm \mu^\mp$	$< 2.7 \times 10^{-7}$	$\lvert\epsilon_{e\mu}\rvert < 0.21$
$B^+ \to K^*(892)^+ e^+ \mu^-$	$< 1.3 \times 10^{-6}$	$\lvert\epsilon_{e\mu}\rvert < 7.1 \times 10^{-2}$
$B^+ \to K^*(892)^+ e^- \mu^+$	$< 9.9 \times 10^{-7}$	$\lvert\epsilon_{e\mu}\rvert < 6.2 \times 10^{-2}$
$B^+ \to K^*(892)^+ e^\pm \mu^\mp$	$< 1.4 \times 10^{-7}$	$\lvert\epsilon_{e\mu}\rvert < 1.7 \times 10^{-2}$
$B^0 \to K^*(892)^0 e^+ \mu^-$	$< 5.3 \times 10^{-7}$	$\lvert\epsilon_{e\mu}\rvert < 4.5 \times 10^{-2}$
$B^0 \to K^*(892)^0 e^- \mu^+$	$< 3.4 \times 10^{-7}$	$\lvert\epsilon_{e\mu}\rvert < 3.6 \times 10^{-2}$
$B^0 \to K^*(892)^0 e^\pm \mu^\mp$	$< 5.8 \times 10^{-7}$	$\lvert\epsilon_{e\mu}\rvert < 3.4 \times 10^{-2}$
$K^+ \to \pi^+ e^+ \mu^-$	$< 1.3 \times 10^{-11}$	$\lvert\epsilon_{e\mu}\rvert < 0.44$ [0.8]
$K^+ \to \pi^+ e^- \mu^+$	$< 5.2 \times 10^{-10}$	$\lvert\epsilon_{e\mu}\rvert \sim O(1)$
$K_L \to \pi^0 e^\pm \mu^\mp$	$< 6.2 \times 10^{-9}$	$\lvert\epsilon_{e\mu}\rvert \sim O(1)$

Table 3. Constraints from $M \to M' l \bar{l}'$.

where τ_K is the lifetime of the Kaon. Note that the model-dependent form factors do not appear in the above formulas. Using the experimental value $B(K^+ \to \pi^0 e^+ \nu_e) = (5.08 \pm 0.05)\%$ [12], we obtain the constraints on $\epsilon_{ll'}$ shown in table 3. Alternatively, the decay rate for $K \to \pi l \bar{l}'$ can be calculated by using eq. (4.7). In this case, the mass of muon is not neglected and the parameter $\rho = m_\mu/(2q^2)$. The relevant form factors are given by

$$f_+^{K\pi}(q^2) \simeq -1 - \lambda_+ q^2 \,,$$
$$\tilde{f}_0^{K\pi}(q^2) \equiv f_+^{K\pi}(q^2) + \frac{q^2}{m_K^2 - m_\pi{}^2} f_-^{K\pi}(q^2) \simeq -1 - \lambda_0 q^2 \,, \tag{4.11}$$

where $\lambda_+ = 0.067$ fm^2 and $\lambda_0 = 0.040$ fm^2 [20]. The constraints on $\epsilon_{ll'}$ obtained in this way (number shown in the bracket for $K^+ \to \pi^+ e^+ \mu^-$) is similar to those obtained by using eq. (4.10) as shown in table 3. The constraints obtained here are again much weaker than those obtained from $\tau \to Pl$.

5 Constraint from muonium-antimuonium oscillation

At tree level, exchange of Z boson can generate an effective Hamiltonian of the form

$$H_{eff} = \sqrt{2} G_F \epsilon_{e\mu}^{*2} \bar{\mu} \gamma_\mu (1 - \gamma_5) e \bar{\mu} \gamma^\mu (1 - \gamma_5) e \,. \tag{5.1}$$

This interaction will result in muonium-antimuonium oscillation.

Process	Conversion rate	Constraint on $\lvert\epsilon_{ll'}\rvert$
$\mu - e$ conversion	$< 4.3 \times 10^{-12}$	$\lvert\epsilon_{e\mu}\rvert < 1.7 \times 10^{-7}$
Process	Branching Ratio	Constraint on $\lvert\epsilon_{ll'}\rvert$
$\mu^- \to e^+e^-e^-$	$< 1 \times 10^{-12}$	$\lvert\epsilon_{e\mu}\rvert < 1.1 \times 10^{-6}$
$\tau^- \to e^+e^-e^-$	$< 3.6 \times 10^{-8}$	$\lvert\epsilon_{e\tau}\rvert < 5.1 \times 10^{-4}$
$\tau^- \to \mu^+\mu^-\mu^-$	$< 3.2 \times 10^{-8}$	$\lvert\epsilon_{\mu\tau}\rvert < 4.9 \times 10^{-4}$
$\tau^- \to \mu^+\mu^-e^-$	$< 4.1 \times 10^{-8}$	$\lvert\epsilon_{e\tau}\rvert < 7.2 \times 10^{-4}$
$\tau^- \to e^+e^-\mu^-$	$< 2.7 \times 10^{-8}$	$\lvert\epsilon_{\mu\tau}\rvert < 5.6 \times 10^{-4}$
$\mu^- \to e\gamma$	$< 1 \times 10^{-15}$	$\lvert\epsilon_{e\mu}\rvert \lesssim 1.1 \times 10^{-4}$
$\tau^- \to e\gamma$	$< 5 \times 10^{-11}$	$\lvert\epsilon_{e\tau}\rvert \lesssim 2.4 \times 10^{-2}$
$\tau^- \to \mu\gamma$	$< 4 \times 10^{-11}$	$\lvert\epsilon_{\mu\tau}\rvert \lesssim 1.5 \times 10^{-2}$

Table 4. Constraints from $l_i \to l_j\bar{l}_k l_l$, $l_i \to l_j\gamma$ decays and $\mu - e$ conversion.

Process	Branching Ratio	Constraint on $\lvert\epsilon_{ll'}\rvert$
$Z \to e^\pm\mu^\mp$	$< 1.7 \times 10^{-6}$	$\lvert\epsilon_{e\mu}\rvert < 1.8 \times 10^{-3}$
$Z \to e^\pm\tau^\mp$	$< 9.8 \times 10^{-6}$	$\lvert\epsilon_{e\tau}\rvert < 4.3 \times 10^{-3}$
$Z \to \mu^\pm\tau^\mp$	$< 1.2 \times 10^{-5}$	$\lvert\epsilon_{\mu\tau}\rvert < 4.8 \times 10^{-3}$

Table 5. Constraints on $\epsilon_{ll'}$ from $Z \to l\bar{l}'$ decays.

The SM prediction for muonium and antimuonium oscillation is extremely small. Observation of this oscillation at a substantially larger rate will be an indication of new physics. Experimentally, no oscillation has been observed. The current upper limit for the probability of spontaneous muonium to antimuonium conversion was established at $P_{\bar{M}M} \leq 8.3 \times 10^{-11}$ (90% C.L.) in 0.1 T magnetic field [21].

In the absence of external electromagnetic fields, the probability $P_{\bar{M}M}$ of observing a transition can be written as [22] $P_{\bar{M}M}(0\mathrm{T}) \simeq |\delta|^2/(2\Gamma_\mu^2)$, where $\delta \equiv 2\langle\bar{M}|H_{eff}|M\rangle$ and Γ_μ is the muon decay width. For H_{eff} given above, the transition amplitude is given by $\delta = 32G_F\epsilon_{e\mu}^2/(\sqrt{2}\pi a^3)$ for both triplet and singlet muonium states, where $a \simeq (\alpha m_e)^{-1}$ is the Bohr radius. The probability $P_{\bar{M}M}$ has strong magnetic field dependence which usually occurs in experimental situation. With an external magnetic field, there is a reduction factor S_B, i.e. $P_{\bar{M}M}(B) = S_B P_{\bar{M}M}(0\mathrm{T})$. The magnetic field correction factor S_B describes the suppression of the conversion in the external magnetic field due to the removal of degeneracy between corresponding levels in $\bar{M}$ and M. One has $S_B = 0.35$ for our case at $B = 0.1\mathrm{T}$ [21, 23]. Using this experimental information, one obtains a constraint

$$|\epsilon_{e\mu}| < 4 \times 10^{-2}. \tag{5.2}$$

This constraint is rather weak compared with that from $\mu - e$ conversion.

Exchange of Higgs boson will also contribute. But this contribution is suppressed by a factor m_μ^2/m_h^2 and can be safely neglected compared with Z boson exchange contribution.

JHEP09(2009)027

6 Constraints from $l_i \to l_j \bar{l}_k l_l$, $l_i \to l_j \gamma$, $Z \to l\bar{l}'$ decays and $\mu - e$ conversion

These processes have been studied in the literature before [9, 10]. For comparison, we summarize the results for constraints on $\epsilon_{ll'}$ for $l_i \to l_j \bar{l}_k l_l$, $l_i \to l_j \gamma$ and $\mu - e$ conversion, and $Z \to l\bar{l}'$[1] in tables 4 and 5, respectively. The most stringent upper bound on $|\epsilon_{e\mu}|$ is of order 10^{-7} from $\mu - e$ conversion in atomic nuclei. The upper bounds on $|\epsilon_{e\tau}|$ and $|\epsilon_{\mu\tau}|$ obtained are of order 10^{-4} from $\tau \to e\bar{e}e$ and $\tau \to \mu\bar{\mu}\mu$ decays.

7 Discussions on the mixing matrix $U_{\nu N}$ between the light and heavy neutrinos

We now discuss some implications of the constraints obtained earlier on the model parameters. In this model, to the order we are studying, the light neutrino mass is related to $U_{\nu N}$ with

$$U_{\rm PMNS} \hat{m}_\nu U_{\rm PMNS}^T = -U_{\nu N} M_\Sigma U_{\nu N}^T \ , \tag{7.1}$$

where the light neutrino mass matrix $\hat{m}_\nu$ is diagonal:

$$\hat{m}_\nu = {\rm diag}\,(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) = U_{\rm PMNS}^\dagger m_\nu U_{\rm PMNS}^* \ . \tag{7.2}$$

Thus, one might think that the elements of $U_{\nu N}$ are too small to be relevant to the FCNC discussion, because with only one generation of the light and heavy neutrinos, $|U_{\nu N}|$ is simply given by $(m_\nu/M_\Sigma)^{1/2}$. It leads to the fact that for $M_\Sigma > 100\,{\rm GeV}$, $U_{\nu N}$ is less than 10^{-5}, since the light neutrino masses must be less than an eV or so. If with more than one generation of the light and heavy neutrinos, all elements of $U_{\nu N}$ are the same order of magnitudes (the canonical seesaw models), the resulting elements of the ϵ matrix will all be way below the constraints we have obtained. This makes the model irrelevant for an experimental detection. The FCNC study of the kind studied here is therefore not interesting for canonical seesaw models. However, it has been shown that with more than one generation of the light and heavy neutrinos, there are non-trivial solutions of $U_{\nu N}$ such that the right hand side of eq. (7.1) becomes exactly zero but the elements of $U_{\nu N}$ can be arbitrarily large [24, 25]. Thus, these solutions evade the canonical seesaw constraint $|U_{\nu N}| = (m_\nu/M_\Sigma)^{1/2}$ held in the one generation case [24, 25]. It is interesting if one can find the $U_{\nu N}$ which satisfies existing experimental constraints by adding small perturbations to the above non-trivial solutions. A recent study has shown such solutions of $U_{\nu N}$ that indeed can have large elements and satisfy the current experimental constraints [25]. In the following we will describe some of those solutions having relevance to our FCNC study.

Let us indicate the solution of $U_{\nu N}$ which gives zero light neutrino mass as U_0. We then add a perturbation U_δ to U_0 such that $U_{\nu N} = U_0 + U_\delta$. Since $U_0 M_\Sigma U_0^T = 0$, the

[1] The numerical values of the bounds shown in table 5 are slightly smaller than those in ref. [10]. It is because the experimental bounds on the branching ratios are given for the *sum* of the charge states or particle/antiparticle states indicated [12] so that the decay rate for $Z \to l\bar{l}'$ used in [10] must be *doubled* when one uses the experimental results to obtain the constraints on $|\epsilon_{ll'}|$.

JHEP09(2009)027

neutrino mass matrix is given by

$$m_\nu = -U_0 M_\Sigma U_\delta^T - U_\delta M_\Sigma U_0^T - U_\delta M_\Sigma U_\delta^T \ . \tag{7.3}$$

If the first two terms are not zero, the matrix elements a_{ij} in U_0 and δ_{ij} in U_δ are of order $a_{ij}\delta_{ij} \sim m_\nu/M_\Sigma$ which is much smaller than 1. Since we are interested in having large a_{ij}, the elements δ_{ij} must be much smaller than a_{ij}, and the third term, for practical purpose, can be neglected. If on the other hand, the first two terms are zero, the third term must be kept. The elements of U_δ in this case are of order $(m_\nu/M_\Sigma)^{1/2}$.

In the basis where M_Σ is diagonal, one can write

$$M_\Sigma \ = \ \hat{M}_\Sigma \ = \ \mathrm{diag}\left(\frac{1}{r_1}, \frac{1}{r_2}, \frac{1}{r_3}\right) m_N \ , \qquad r_l \ = \ \frac{m_N}{M_l} \ , \tag{7.4}$$

where, for convenience, we have introduced a scale parameter m_N to represent the scale of the heavy neutrino, which we choose to be the lightest of the heavy neutrinos. The contribution to ϵ is given by

$$\epsilon = U_{\nu N} U_{\nu N}^\dagger \approx U_0 U_0^\dagger \ . \tag{7.5}$$

We show three types of solutions relevant to our study of FCNC: (a) sizeable $\epsilon_{12,13,23}$; (b) sizeable ϵ_{23} and small $\epsilon_{12,13}$; and (c) sizeable ϵ_{13} and small $\epsilon_{12,23}$. In case (a), the data from $\mu - e$ conversion in atomic nuclei constrain $|\epsilon_{12}|$ to be less than 1.7×10^{-7} which makes $\epsilon_{13,23}$ too small to be of interest. We therefore need to find other classes of solutions where ϵ_{12} is automatically much smaller than $\epsilon_{13,23}$. These are the cases (b) and (c). If these types of solutions are correct, the constraints from τ decays discussed previously in this paper are still relevant for experimental search.

The numerical results will be given by using the central values of $\Delta m^2_{21} = \left(7.65^{+0.23}_{-0.20}\right) \times 10^{-5}\,\mathrm{eV}^2$ and $\left|\Delta m^2_{31}\right| = \left(2.40^{+0.12}_{-0.11}\right) \times 10^{-3}\,\mathrm{eV}^2$, determined by a recent fit to global neutrino data [26], and U_{PMNS} in the tri-bimaximal form [27] for simplicity

$$U_{\mathrm{tribi}} \ = \ \begin{pmatrix} \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \end{pmatrix} . \tag{7.6}$$

For the details of the following solutions, we refer to ref. [25].

For case (a), a desired solution is given by

$$U_0^a = U_{\mathrm{PMNS}} \begin{pmatrix} a & a & i\sqrt{2}a \\ b & b & i\sqrt{2}b \\ c & c & i\sqrt{2}c \end{pmatrix} \mathcal{R} \ , \quad U_\delta^a = U_{\mathrm{PMNS}} \begin{pmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{pmatrix} \mathcal{R} \, , \tag{7.7}$$

where $\mathcal{R} = \mathrm{diag}\left(\sqrt{r_1}, \sqrt{r_2}, \sqrt{r_3}\right)$. There are two types of solutions corresponding to normal and inverted hierarchies in light neutrino masses, but always one of the masses becomes zero as follows.

JHEP09(2009)027

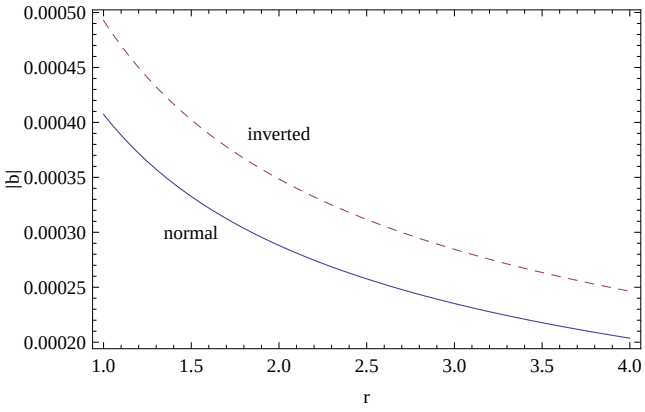

Figure 1. For case (a), the upper limits on the magnitude of the element of $U_{\nu N}$ in terms of the heavy neutrino mass parameter $r \equiv r_1 + r_2 + 2r_3$. The solid and dashed lines correspond to the normal and inverted hierarchy cases, respectively.

(i) Normal hierarchy:

$$a = 0, \qquad \hat{m}_\nu = \text{diag}\left(0,\, -1,\, \frac{c^2}{b^2}\right) 2\tilde{\delta} b m_N \,,$$
$$\epsilon = \begin{pmatrix} 0.33 & 0.33 - 0.97\,i & 0.33 + 0.97\,i \\ 0.33 + 0.97\,i & 3.1 & -2.5 + 1.9\,i \\ 0.33 - 0.97\,i & -2.5 - 1.9i & 3.1 \end{pmatrix} |b|^2\, r \,, \tag{7.8}$$

(ii) Inverted hierarchy:

$$c = 0, \qquad \hat{m}_\nu = \text{diag}\left(\frac{a^2}{b^2},\, -1,\, 0\right) 2\tilde{\delta} b m_N \,,$$
$$\epsilon = \begin{pmatrix} 0.99 & 0.01 - 0.70\,i & 0.01 - 0.70\,i \\ 0.01 + 0.70\,i & 0.50 & 0.50 \\ 0.01 + 0.70\,i & 0.50 & 0.50 \end{pmatrix} |b|^2\, r \,, \tag{7.9}$$

where $\tilde{\delta} = \delta_{21} + \delta_{22} + i\sqrt{2}\delta_{23}$ and $r = r_1 + r_2 + 2r_3$. From $\mu - e$ conversion in atomic nuclei $\left(|\epsilon_{12}| = |\epsilon_{e\mu}| < 1.7 \times 10^{-7}\right)$, $|b|\sqrt{r}$ is constrained to be smaller than 4.1×10^{-4} (normal hierarchy) or 4.9×10^{-4} (inverted hierarchy). In both cases, $|\epsilon_{13,23}|$ are constrained to be less than $O(10^{-7})$ which are way below the best constrained from $\tau \to \mu\bar{\mu}\mu$ and $\tau \to \pi^0 e$ decays. In figure 1 we show the upper limits from the $\mu - e$ conversion constraint on the magnitude of the element b of $U_{\nu N}$ in terms of the heavy neutrino mass parameter r. Since m_N is the lightest of M_l, r is in the range $1 \le r \le 4$. Depending on the heavy neutrino mass hierarchy, the value of $|b|$ can be different. With the same constraint, to have the largest b, one would require the two heavier ones to be much larger than the lightest m_N. As far as FCNC processes are concerned, the hierarchy of the heavy neutrinos is not important because the parameter always involves r. But for the production of a heavy lepton at LHC, via $q\bar{q}' \to W^* \to lN$ or $q\bar{q} \to (Z^*, h^*) \to lE$ for example, it is preferred to have a larger

JHEP09(2009)027

b, because in that case, not the combination $|b|^2(r_1+r_2+2r_3)$ but the individual $|b|^2 r_j$ is relevant to the production cross section.

For case (b), the following form serves the purpose with the choice $U_{\nu N} = U_0^b + U_{\alpha\beta\gamma}^b + U_\delta^b$, where

$$U_0^b \;=\; \begin{pmatrix} 0 & 0 & 0 \\ 0 & a & ia \\ 0 & b & ib \end{pmatrix}\mathcal{R}\,, \quad U_{\alpha\beta\gamma}^b \;=\; \begin{pmatrix} \alpha & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\mathcal{R}\,, \quad U_\delta^b \;=\; \begin{pmatrix} 0 & \delta_{12} & 0 \\ 0 & \delta_{22} & 0 \\ 0 & \delta_{32} & 0 \end{pmatrix}\mathcal{R}\,. \tag{7.10}$$

Here α is of order $[(a,b)\delta_{ij}]^{1/2}$ so that one should keep α^2 terms in the calculation, neglecting $\delta_{ij}\delta_{kl}$ and $\alpha\delta_{ij}$ terms. The eigen-masses are

$$\hat{m}_\nu \;=\; \mathrm{diag}\big(a\,\delta_{12}-\alpha^2,\, -2a\,\delta_{12}-\alpha^2,\, 0\big)m_N\,, \tag{7.11}$$

and so this is an inverted hierarchy case with $m_{\nu_3}=0$. Numerically, the matrix ϵ is given by

$$\epsilon \;=\; \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}|a|^2\,\rho\,, \tag{7.12}$$

where $\rho = r_2 + r_3$. Thus, the constraint $|\epsilon_{23}| = |\epsilon_{\mu\tau}| < 4.9\times10^{-4}$ from $\tau\to\mu\bar{\mu}\mu$ decays translates into $|a|\sqrt{\rho} < 2.2\times10^{-2}$. Since r_1 does not show up in U_0^b in this case, it would be more convenient to choose m_N to be the lightest of $M_{2,3}$.

For case (c), the desired results can be obtained by choosing $U_{\nu N} = U_0^c + U_{\alpha\beta\gamma}^c + U_\delta^b$, with

$$U_0^c \;=\; \begin{pmatrix} 0 & a & ia \\ 0 & 0 & 0 \\ 0 & b & ib \end{pmatrix}\mathcal{R}\,, \quad U_{\alpha\beta\gamma}^c \;=\; \begin{pmatrix} \alpha & 0 & 0 \\ \beta & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\mathcal{R}\,. \tag{7.13}$$

This particular choice allows all the three light-neutrinos to have nonzero masses. Taking $m_{\nu_2} = 0.1\,\mathrm{eV}$, two possible solutions are found and give the matrix ϵ as follows.

(i) Normal hierarchy $\big($with $m_{\nu_1} = 0.0996\,\mathrm{eV}$ and $m_{\nu_3} = 0.111\,\mathrm{eV}\big)$:

$$\epsilon \;=\; \begin{pmatrix} 1 & 0 & 0.001-1.0\,i \\ 0 & 0 & 0 \\ 0.001+1.0\,i & 0 & 1.1 \end{pmatrix}|a|^2\,\rho\,, \tag{7.14}$$

(ii) Inverted hierarchy $\big($with $m_{\nu_1} = 0.0996\,\mathrm{eV}$ and $m_{\nu_3} = 0.0867\,\mathrm{eV}\big)$:

$$\epsilon \;=\; \begin{pmatrix} 1 & 0 & 0.001+0.96\,i \\ 0 & 0 & 0 \\ 0.001-0.96\,i & 0 & 0.93 \end{pmatrix}|a|^2\,\rho\,. \tag{7.15}$$

JHEP09(2009)027

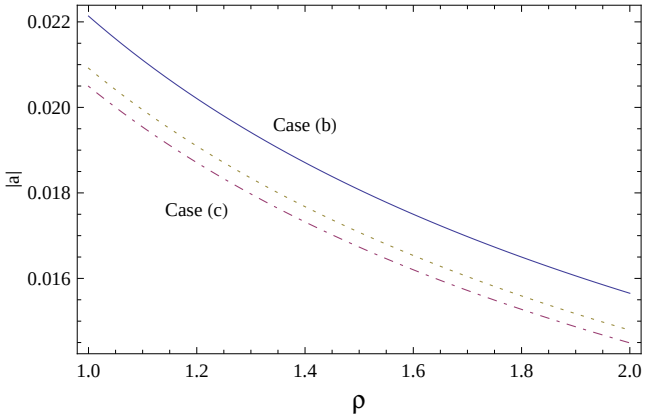

Figure 2. For cases (b) and (c), the upper limits on the magnitude of the element of $U_{\nu N}$ in terms of the heavy neutrino mass parameter $\rho \equiv r_2 + r_3$. The solid line corresponds to case (b) and the dot-dashed and dotted lines correspond to the normal and inverted hierarchy cases, respectively, in case (c).

The bound $|\epsilon_{13}| = |\epsilon_{e\tau}| < 4.2 \times 10^{-4}$ from $\tau \to \pi^0 e$ decays then implies $|a|\sqrt{\rho} < 2.0 \times 10^{-2}$ in the two cases. In figure 2 we display the upper limits on the magnitude of the element a of $U_{\nu N}$ in terms of the heavy neutrino mass parameter ρ for cases (b) and (c). In this case, ρ is in the range $1 \leq \rho \leq 2$. With the same constraint, the hierarchy that the heavier of $M_{2,3}$ is much larger than m_N would be required to obtain the largest a. Similarly to case (a), concerning FCNC processes, the hierarchy of the heavy neutrinos is not important. But, concerning the production of a heavy lepton N or E at LHC, a large cross section can be obtained for $m_N \lesssim 115\,\text{GeV}$ [25].

The above examples clearly show that with the constraints from FCNC transitions as well as from the tiny neutrino masses, the elements of $U_{\nu N}$ can still be large. There is another class of processes which also provides constraints on the elements of $U_{\nu N}$. These processes involve neutral currents conserving lepton flavor and can be used to test deviations from the SM predictions for electroweak precision data (EWPD) [28]. They have been measured mainly at LEP and provide bounds on the combinations of the diagonal elements of $U_{\nu N}$. The constraints extracted from the EWPD are $|(U_{\nu N})_{ii}| \leq \mathcal{O}(0.01)$ [28]. In contrast, the FCNC constraints discussed above involve combinations containing the off-diagonal elements and impose more stringent constraints, such as $|\epsilon_{12}| = \left|\sum_k (U_{\nu N})_{1k}(U^*_{\nu N})_{2k}\right| < 1.7 \times 10^{-7}$. The non-zero elements of $U_{\nu N}$ in the two examples we give above with suppressed ϵ_{12}, being at most of $\mathcal{O}(0.01)$, satisfy all these constraints.

Large elements of $U_{\nu N}$ also have important implications for a direct test of the model by producing the heavy neutrinos at LHC. The elements of $U_{\nu N}$ with the magnitude of order 0.01 are large enough to be detectable at LHC [25]. The heavy neutrino N can be produced through the mixing via $q\bar{q}' \to W^* \to l^\pm N$. Similarly, the heavy charged lepton E can also be produced through the mixing via $q\bar{q} \to (Z^*, h^*) \to l^\pm E^\mp$ and $q\bar{q}' \to W^* \to \nu E^\pm$. At LHC the production cross section for a single heavy neutrino N can be larger than 1 fb if the heavy neutrino mass is less than 115 GeV with the elements of $U_{\nu N}$ being 0.01.

JHEP09(2009)027

The production cross section of a single E is slightly smaller. This can provide useful information about this model.

8 Conclusions

We have systematically studied various FCNC processes in the lepton sector in the framework of Type III seesaw model. Using the current experimental results, we have put the constraints on the parameters $\epsilon_{ll'}$ which are responsible for tree level FCNC in the charged lepton sector. The new processes that have been considered are: the LFV processes $\tau \to Pl$, $\tau \to Vl$, $V \to l\bar{l}'$, $P \to l\bar{l}'$, $M \to M'l\bar{l}'$ and muonium-antimuonium oscillation.

Although exchange both Z and Higgs bosons at tree level can induce FCNC in charged lepton sector, we find that there is no contribution from Higgs exchange in the processes $\tau \to P(V)l$ and $V(P) \to l\bar{l}'$, and the effects of Higgs exchange are negligibly small in the last two classes of processes.

We now compare constraints on various FCNC parameters obtained from processes considered in this work with those obtained in previous studies. It turns out that with currently available experimental data, the LFV processes considered in this work involving τ leptons provide very stringent constraints on the FCNC parameter $\epsilon_{i\tau}$. Our results show that the most stringent constraint on $\epsilon_{e\tau}$ comes from $\tau \to \pi^0 e$ decay with $|\epsilon_{e\tau}| < 4.2 \times 10^{-4}$. $\tau \to \rho^0 \mu$ and $\tau \to \pi^0 \mu$ give very stringent constraints on $\epsilon_{\mu\tau}$ with $|\epsilon_{\mu\tau}| < 6.8 \times 10^{-4}$ and $|\epsilon_{\mu\tau}| < 7.0 \times 10^{-4}$, respectively, comparable with $|\epsilon_{\mu\tau}| < 4.9 \times 10^{-4}$ obtained from $\tau \to \mu\bar{\mu}\mu$ in previous studies. The strongest constraint on $\epsilon_{e\mu}$ comes from $\mu - e$ conversion in atomic nuclei studied previously with $|\epsilon_{e\mu}| < 1.7 \times 10^{-7}$. The new constraint on $\epsilon_{e\mu}$ obtained from processes considered in this work is much weaker.

Two body meson decays, such as $\Upsilon(3S) \to l\bar{l}'$, $J/\Psi \to l\bar{l}'$, $\pi \to l\bar{l}'$ and $\eta^{(\prime)} \to l\bar{l}'$, provide rather weak bounds on $|\epsilon_{ll'}|$ at most of order 10^{-1}. The constraints from semileptonic three body B or K decays of the type $M \to M'l\bar{l}'$ are also rather weak with upper bounds on $|\epsilon_{ll'}|$ in the range $O(10^{-2}) \sim O(1)$.

In the canonical seesaw models, where the elements of $U_{\nu N}$ are of the same order of magnitude as that for an one generation seesaw model, $(m_\nu/m_N)^{1/2}$, it is not possible to have elements of ϵ which are sufficiently large to reach the FCNC bounds studied in this paper. The FCNC effects studied are therefore not interesting for the canonical seesaw models. However, with more than one generation of light and heavy neutrinos, in certain special circumstances the mixing is not constrained directly by the tiny neutrino masses and therefore can be large. Thus in this class of seesaw models, it is possible to have large FCNC interactions. These circumstances have been studied by several groups [24, 25]. We find some example solutions which can lead to the FCNC parameters ϵ_{ij} large enough to reach the constraints obtained here. The search for FCNC effects can still provide further information on the seesaw models. We comment that efforts in constructing models with certain symmetries to evade the canonical seesaw constraints on the mixing matrix $U_{\nu N}$ have been made in various ways [29]. It would be interesting to further study related phenomenology to test these models.

JHEP09(2009)027

We would like to comment that in some processes considered in this work it is possible to have CP violating signatures, such as lepton and anti-lepton decay rate asymmetries, and asymmetries in Z decays into $\bar{l}l'$ and $\bar{l}'l$ [30]. To have non-zero effects, one needs not only a weak phase appearing in CP violating couplings coming from the complex ϵ_{ij} and U_{PMNS} matrix, but also a strong phase appearing in an absorptive part from loop induced decay amplitudes. Since we consider that the heavy seesaw scale M is heavier than Z, no absorptive part will be developed with the heavy triplets in the loop. Only light degrees of freedom in the loop for Z decays into $\bar{l}l'$ and $\bar{l}'l$ can generate the absorptive parts which are generally small. The resulting CP violating effect will therefore be small. If polarizations of the initial and final particles can be measured, it is possible to construct CP violating observables which does not need the absorptive parts [31]. We will carry out the detailed studies elsewhere.

Finally let us comment on several possible improvements on $\epsilon_{ll'}$ from future experiments. Improved data for $\tau \to Pl$ and $\tau \to Vl$ decays at various facilities, such as B and τ-Charm factories, can improve the bounds on $\epsilon_{ll'}$. Bounds from $V \to l\bar{l}'$ and $P \to l\bar{l}'$ can also be improved, but may not be able to compete with constraints from other experiments. The current bound from $B \to K\mu\tau$ is rather weak. But at LHCb about 10^{12} $b\bar{b}$ pairs are expected to be produced each year, and this decay mode may be useful in improving bound on $\epsilon_{\mu\tau}$. Rare kaon decays will be studied at J-PARC with high precisions so that the current weak bounds from kaon decays may also become much stronger. But bounds obtained may still not be competitive with others. $\mu - e$ conversion in atomic nuclei will also be studied at J-PARC with several orders of magnitude improvement in sensitivity. Constraint on $\epsilon_{e\mu}$ can be improved by more than an order of magnitude. It may be very difficult to improve constraint on $\epsilon_{e\mu}$ from muonium-antimuonium oscillation to the level $\mu - e$ conversion can achieve. At present $Z \to l\bar{l}'$ do not provide the best bounds on $\epsilon_{ll'}$. However, the Giga-Z modes at future colliders, such as ILC, the sensitivity can be improved by up to three orders of magnitudes [32]. Future studies of $Z \to e\tau$ and $Z \to \mu\tau$ may improve the bounds on $\epsilon_{e\tau}$ and $\epsilon_{\mu\tau}$. It is clear that FCNC effects in Type III seesaw model can be further tested.

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JHEP09(2009)027